Self-Reference and Time According to G. Spencer-Brown *

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1 Introduction

Self-reference is ubiquitous. It occurs in any comprehensive description of the universe. It is found in neuroscience where the brain studies itself, in attempts to describe the biological characteristics of life [4], in mathematics when dealing with questions of provability [1], in sociology when describing the society from within [3], and in the physics of quantum cosmology [7]. However, self-reference is irritating, too. It leads to paradoxes and antinomies, to propositions equivalent to their own negation. The difficulties related to self-reference seem to be rooted in language. Thus, despite its ubiquity, self-reference in everyday life is neither a topic of practical interest nor does it present a problem. But why this is the way it is?

An attempt to clarify the problem of self-reference on a fundamental level has been made by George Spencer-Brown [5]. In his book 'Laws of Form' George Spencer-Brown presents a calculus dealing with self-reference without running into inevitable antinomies. The main feature of interest here is the occurrence of a thight connection of self-referential and temporal structures leading to the emergence of time out of self-reference.

2 Laws of Form

The basic idea of the 'Laws of Form' is that any description presupposes a fundamental distinction which is found for e.g. in the difference between description and its subject as well as in the difference between the observer and the observed. The 'Laws of Form' develop a formal system called the Calculus of Indications whose fundamentals are not only in agreement with but are restricted to this presupposition. As a consequence, the

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calculus may be considered as underlying every attempt to describe any universe. As far as an universe is inaccessible without a description and every description presupposes a distinction,

> ... a universe comes into being when a space is severed[1] ... .[2]

In analogy, the development of the calculus starts with an injunction to make a distinction, the first distinction. The laws governing the form of this first as well as every other distinction are the topic of the 'Laws of Form'. These laws are based on a difference between the descriptive and injunctive aspect of indications as expressed in the two axioms [5]

> *Axiom 1.* The law of calling
> *The value of a call made again is the value of the call.*
> That is to say, if a name is called and then called again, the value indicated by the two calls taken together is the value indicated by one of them.
> That is to say, for any name, to recall is to call.
>
> *Axiom 2.* The law of crossing
> *The value of a crossing made again is not the value of the crossing.*
> That is to say, if it is intended to cross a boundary and then it is intended to cross it again, the value indicated by the two intentions taken together is the value indicated by non of them.
> That is to say, for any boundary, to recross is not to cross.

The attempt undertaken in the 'Laws of Form' goes beyond the concept of truth and logic. It is intended to reach out for the common ground of both, the logic and its subject, the structure of any universe. Thus, it provides a foundation for a theory of structures in general, including, by the capability of the Calculus of Indications to deal with self-reference, self-referential structures. The manifestations of self-referential structures recognized within the calculus leads to a deeper insight into the problem of self-reference, whether related to logic, language or to the observer itself.

3 The Calculus of Indications

3.1 Distinction and Indication

The starting point of the *calculus of indication*[3] is given by the the *idea of distinction* and the *idea of indication*. Both concepts are related by *the form* of the distinction which itself is considered as the fundamental structure. The form represents the distinction and

[3] For exact definitions of terms printed in italic c.f. 'Laws of Form' [5].

its subject but without a reference to either side of the distinction. However, there is no factual distinction without an indication of one side of the distinction as well as there are no factual sides without a distinction. Thus, the act of making a distinction involves both the idea of indication and the idea of distinction in a complementary way. When referring to *the first distinction* the reader is advised to make, the form becomes, by the act of making this first distinction, realized. The indicated part of the first distinction is known as the *marked state*, marked by *the mark* $\urcorner$. In a schematic way this may be represented as shown in figure 1.

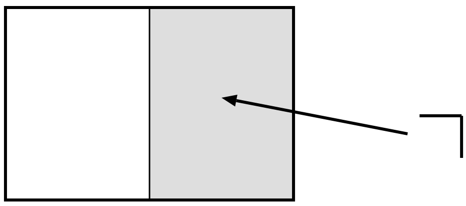

Figure 1: Schematic representation of the first distinction, the marked state and its indication.

What is essential about the mark is that the mark itself is considered as distinction. It has an outside and an inside as indicated in figure 2. The two-fold meaning of $\urcorner$ allows a definition of a calculus without introducing the ordinary operator-operand distinction.

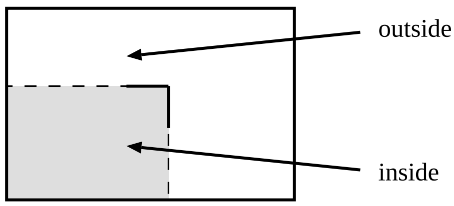

Figure 2: The inside and the outside of the mark.

The initials of the Calculus of Indications are

$$\urcorner\;\urcorner \;=\; \urcorner\;. \tag{1}$$

$$\overline{\urcorner}\!\rceil \;=\; \quad . \tag{2}$$

where the blank on the right hand side of the equation (2) is associated with the *unmarked state*. The two initial equations refer to the two-fold possibility of establishing or making a distinction by either a descriptive or an injunctive approach to indicate the marked state. The descriptive approach indicates the marked state by *calling* it by its name $\overline{\;\;}|$ while the injunctive approach indicates the marked state by interpreting $\overline{\;\;}|$ as a injunction to *cross* the boundary towards it. The initials (1) and (2) reflect in a formal way the different repetitive behavior of the two aspects of indications as expressed in the two axioms of the 'Laws of Form'. The two-fold structure of the indication lies at the very heart of the Calculus of Indications. It is intended as a comprehensive criticism to the sentence 7 of Wittgenstein's Tractatus [8]

> Wovon man nicht sprechen kann, darüber muss man schwiegen.

which may be interpreted to refer only to the descriptive aspect of indications.

3.2 The Primary Arithmetic

The *primary arithmetic* exclusively deals with the first distinction made by the reader following the injunction at the starting point of the development of the Calculus of Indications. The primary arithmetic is limited to the part of the calculus directly and finitely generated form the initials (1) and (2). As its numerical counterpart the primary arithmetic deals with methods, means and theorems of calculations. However, it does not refer to numbers but to the marked and unmarked state.

It follows from the initials (1) and (2) that every arithmetic expression is either equivalent to $\overline{\;\;}|$ or . An example of an arithmetic calculation demonstrating the equivalence of a complex expression and $\overline{\;\;}|$ is

$$\overline{\overline{\;\;}|\;\overline{\;\;}|\;}|\;\overline{\overline{\overline{\;\;}|\;}|\;}|\;\overline{\;\;}| = \overline{\overline{\;\;}|\;}|\;\overline{\overline{\overline{\;\;}|\;}|\;}|\;\overline{\;\;}| = \overline{\overline{\overline{\;\;}|\;}|\;}|\;\overline{\;\;}| = \overline{\;\;}|\;\overline{\;\;}| = \overline{\;\;}| . \tag{3}$$

The demonstration involves first (1), subsequently two times (2) and finally again (1). The two central theorems of the primary arithmetic are

$$\overline{\overline{\mathrm{p}}|\;\;\mathrm{p}\;}| \;=\; \quad . \tag{4}$$

$$\overline{\overline{\mathrm{pr}}|\;\;\overline{\mathrm{qr}}|\;}| \;=\; \overline{\overline{\mathrm{p}}|\;\;\overline{\mathrm{q}}|\;}|\;\mathrm{r} \quad . \tag{5}$$

where p, q and r represent expressions of known value either equivalent to $\overline{\;\;}|$ or .

3.3 The Primary Algebra

The *primary algebra* considers the theorems (5) and (4) taken out of the context of the primary arithmetic as its initials. Within the algebra, p, q and r are variables indicating expressions of unknown value. Thus, the primary algebra is a calculus for the primary arithmetic studying the operations performed on $\overline{\;\;}|$ and , i.e. the operations defined on the ground of the two-fold meaning of $\overline{\;\;}|$ as operator and operand by the initials of the Calculus of Indications (1) and (2). Although both the representations of the primary arithmetic and the primary algebra make use of the same mark $\overline{\ \ }|$, its interpretation changes with the context. The descriptive and injunctive aspect of the arithmetic $\overline{\ \ }|$ in the context of the primary algebra is reduced to the only descriptive nature of the algebraic $\overline{\ \ }|$ indicating the injunctive aspect of the arithmetic $\overline{\ \ }|$.

The theorems of the primary algebra may be classified as either algebraic theorems with an arithmetic or without arithmetic counterpart. Theorems of the last type are purely algebraic. They involve expressions without arithmetic representation as e.g. the infinite expressions

$$\overline{\overline{\overline{\cdots}|}|}|. \tag{6}$$

3.4 Re-Entry

The key feature of the primary algebra is the possibility of re-entering an algebraic expression in itself. The *re-entry* process explicitly reveals the self-referential structure of the calculus of indications and transforms infinite expressions like (6) into a finite form. Expression (6) for e.g. may be considered to be generated by re-entering it in itself according to

$$\mathrm{f1} = \overline{\mathrm{f1}}| \tag{7}$$

$$= \overline{\overline{\overline{\cdots}|}|}| \quad . \tag{8}$$

However, the same expression 6 may also result from

$$\mathrm{f2} = \overline{\overline{\mathrm{f2}}|}| \tag{9}$$

$$= \overline{\overline{\overline{\cdots}|}|}| \quad . \tag{10}$$

and thus

$$\mathrm{f1}\ ,\ \mathrm{f2} = \overline{\overline{\overline{\cdots}|}|}| \quad . \tag{11}$$

In the context of the primary algebra, the equations (7) and (9) are equivalent. In the context of the primary arithmetic, they are not. Arithmetically, equation (9) is fulfilled for $f2 = \overline{\ \ }|$ as well as for $f2 = \quad$. But equation (7) is neither fulfilled for $f1 = \overline{\ \ }|$ nor for $f1 = \quad$. As a consequence, (7) represents arithmetically an inequality. Equation (9) may be considered to be tautological while equation (7) as paradox. Since equation (7) has no solution in the primary arithmetic, the connection of the primary algebra and the primary arithmetic is, as a consequence of the re-entry process, lost.

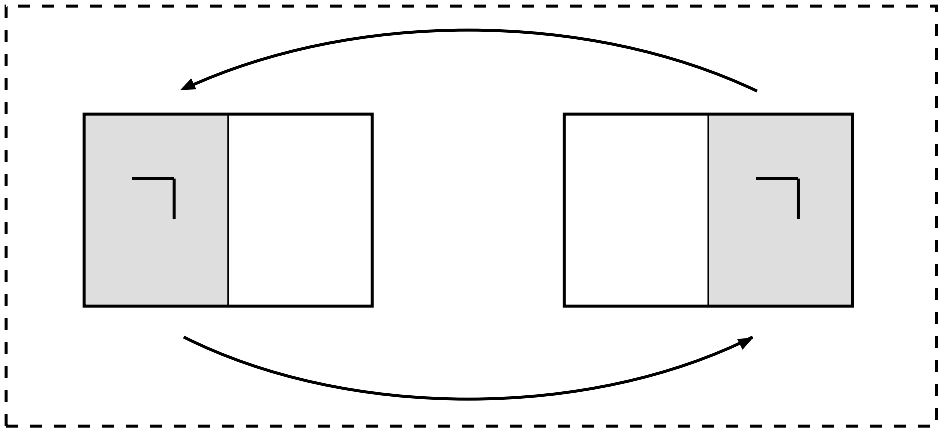

Figure 3: Schematic representation of the imaginary state oscillating in time. It is considered to be the solution of equation (9).

3.5 Self-Reference and Time

By the loss of connection of the primary algebra and the primary arithmetic the Calculus of Indications becomes inconsistent. There are two common possibilities to resolve this situation. First, equation (9) could be excluded from the calculus. This, however, would result in an incompleteness. The second possibility is to include equation (9) as an additional axiom[4].

Spencer-Brown attempt to resolve the problem considers the state described by equation (9) as *imaginary state* alternating between $\urcorner$ and . Thus, if besides[5] $\urcorner$ and   there is no other state and if equation (9) is considered to have a solution, this solution has to be regarded as invoking by its oscillations[6] *time*.

The primary algebra contains besides (6) a variety of other self-referential expressions. By referring to the imaginary state they all are in close connection with the notion of time. It is interesting to note as a side remark that whenever potentially infinite expressions occur in finite form, antinomies are circumvented by the introduction of time. Self-reference, time and finiteness thus seem to belong to one side while infiniteness belongs to another.

To summarize, the self-reference introduced into the calculus by the re-entry process leads to the paradox of equation (9) canceling the connection of the primary algebra and the primary arithmetic. Restoring this connection without changing the domain of the primary arithmetic leads to the inclusion of time.

[4] This line has been explored for e.g. by Varela [6].

[5] The term 'besides' may be taken literally here. The only relation the Calculus of Indications laid so far is the inside-outside relation associated with the mark or the marked-unmarked relation of the first distinction. Both are of the instantaneous, timeless type of side-by-side relations.

[6] Including oscillations brings up a relation of the type before-after (besides the side-by-side relation).

4 The Emergence of Time from the Self-Reference of Distinctions

The fundamental starting point of the Calculus of Indications is the distinction inherent in any description. As a consequence, the Calculus of Indications may be considered as the essence of every description. The self-referential structures encountered within the Calculus of Indications thus correspond to the self-reference occuring ubiquitously in comprehensive attempts to describe for e.g. an universe the observer is belonging to. The Calculus of Indications relates self-reference and time insofar as a consistent picture of self-reference restoring the connection between primary arithmetic and primary algebra cannot be conceived outside time. In analogy, any description involving self-referential structures invokes time.

However, this is only one side. The other side is encountered when contemplating that, in order to be self-consistent, the development of the Calculus of Indications must be based on the calculus itself. Taking the attempt of Spencer-Brown at its most serious, we retrieve the observer, i.e. ourselves dealing with the Calculus of Indications, at the level of the calculus. All the attempts an observer makes in order to deal with the universe and even with a description of it take place in the context of the Calculus of Indications. Thus the structures found within the Calculus of Indications not only comply with the structures of any description but they *are* these structures. As a consequence the observer recovers himself within the calculus and from within the calculus. In this sense:

> We see now that the first distinction ... and the observer are ... identical.[7]

From this point of view the connection between self-reference and time takes on another quality. Time is not explicitly included in the Calculus of Indications. Its occurrence is related to the calculus in a way other findings are as e.g. the position of the observer. Self-reference gives rise to time, or, time emerges out of self-reference. Equation (9) is seen as transcendenting its spatial-like context into temporality. Therefore time appears as a necessary consequence of any description dealing with self-referential structure. Time in particular is a consequence of every attempt heading for a comprehensive description of the universe from within. Thus, time in this sense is not related to the universe itself but to the attempt of its description. In other words, the more or less consistent day-by-day picture we have of ourselves as a part of the universe is necessarily temporal. Its temporality emerges out of considering ourselves as a part (the distinction) of the universe and our consciousness about this (the self-reference).

5 Summary and Conclusions

The Calculus of Indications deals at its arithmetic level with the first distinction. On the algebraic level, the arithmetic itself becomes the subject. Thus, the topic of the primary algebra is distinct from its arithmetic predecessor. Making these distinctions is accompanied by a shift of level. The arithmetical level is immediately related to the first distinction. The primary algebra by contemplating the arithmetics, however, leaves this basic level. Its topic continues to be the first distinction, however mediated by the primary arithmetic now.

As long as only finite non self-referential algebraic expressions are considered, there is a direct representation of the primary arithmetic by the algebra. The loss of connection between primary arithmetic and primary algebra occurs by allowing algebraic expressions to be self-referential. As a consequence, introducing self-reference does not affect the immediate, arithmetic level of the Calculus of Indications but, roughly speaking, the way the primary arithmetic is contemplated. Compared to the common relation between algebraic and arithmetic structures, the relation between the primary arithmetic and the primary algebra of the Calculus of Indications is reversed. The primary arithmetic is indeed primary to the algebra. Changing the domain of the primary arithmetic in order to restore its connection to the algebra would thus result in the context of the 'Laws of Form' as well as in a change of the primary algebra. In order to restore the connection and to retrieve a consistent algebraic, self-reference allowing contemplation of the primary arithmetic, the Calculus of Indications introduces time.

In much the same way as self-reference time is lodged to the primary algebra. Time is not mediated by the primary arithmetic. Thus, it is not to be found on the level of the first distinction. Rather it is related by its algebraic nature to the description of it. By extending the context, time appears to be related to the description of the universe but not to the universe itself. Taking the Calculus of Indications at its most serious, time then is seen as emerging from the attempt to describe the universe form within.

One of the key feature of the notion of time according to the 'Laws of Form' is its algebraic nature. Since the primary algebra contemplates the injunctive aspect of the primary arithmetic, time, also not immediately related to the arithmetic level, reflects its operational aspect. By doing so, time is clearly not ontological.

To recapitulate and summarize, time is seen to emerge form self-referent algebraic expressions contemplating the arithmetic, operational aspect of the Calculus of Indications. According to it, time emerges in its last consequence form self-referential distinctions and indications.

There is a wide variety of conclusions applicable to systems including self-referential structures [4, 3, 7]. First of all, time appears directly related to self-reference. Second, taking the Calculus of Indications at its most serious, time may be considered to emerge from self-reference. At least, the Calculus of Indications provides a strong hint that any description of a system excluding its observer leads necessarily to an atemporal picture tending to antinomies. On the other hand, antinomies may be taken to indicate the implicit presence of time in a description of a system. In the context of Quantum Mechanics this may be seen to confirm the tight connection conjectured between the measurement and the time problem.

However, as a word of caution, one should consider the a possible connection between the Calculus of Indications and e.g. physical theories is far from being obvious. The Calculus of Indications is intended to provide a basis for any description. Therefore one may speculate that a possible way to resolve the problem of time is to reformulate the foundations of e.g. Quantum Mechanics in terms of the 'Laws of Form'.

Another more modest challenge is the comparison of other time concepts to that of the 'Laws of Form'. The Kantian concept of time [2] for e.g. is strictly non ontological. It also possesses in some way a self-referent structure since time is considered to be the form of the inner preception. Thus the time concept of the 'Laws of Form' possesses, roughly speaking, at least two features already known form the Kantian concept of time.

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